Proofs Of Theorems
6 Marks out of 150
(a) Use the diagram below to prove the theorem that states that the line drawn from the centre of a circle and perpendicular to the chord, bisects the chord.

Construction: Join AO and OC

RTP: AM = MC

Proof: In Δ's OAM, OCM
OM is a common side
OA = OC (Radii)
\[ \angle 1 = \angle 2 = 90° \quad \text{(given)} \]
Therefore \[ \triangle OAM \cong \triangle OCM \quad (R, \ H, \ S) \]
Hence AM = MC
(a) Use the diagram below to prove the theorem that states:

"The angle subtended by a chord at the centre of a circle is twice the size of the angle that it subtends at the circle."

\[ \hat{O}_1 = \hat{A} + \hat{B}_1 \quad \text{Exterior angle of triangle} \]

\[ \hat{A} = \hat{B}_1 \quad \text{Isos triangle OR Radii} \]

Similarly in other triangle

\[ \hat{O}_1 = 2 \times \hat{B}_1 \]

\[ \hat{O}_2 = 2 \times \hat{B}_2 \]

Therefore

\[ A\hat{O}C = 2 \times A\hat{B}C \]
Theorem Proof

(a) Use the diagram below to prove the theorem that states that the opposite angles of a cyclic quadrilateral are supplementary.

Draw AO and OC

R.T.P: \( \hat{B} + \hat{D} = 180^\circ \)

Proof:
\[
\hat{O}_2 = 2 \times \hat{B} \quad \text{(Angle at centre)}
\]
\[
\hat{O}_1 = 2 \times \hat{D} \quad \text{(Angle at centre)}
\]
\[
\hat{O}_1 + \hat{O}_2 = 360^\circ
\]
\[
\therefore 2\hat{B} + 2\hat{D} = 360^\circ
\]
\[
\therefore \hat{B} + \hat{D} = 180^\circ
\]
Theorem Proof

(a) Prove the theorem that states the angle between a tangent and a chord is equal to the angle in the alternate segment.

R.T.P: $\angle CAE = \angle ABC$

Construction: refer to diagram for the construction

Proof:

$OA'C + \angle CAE = 90^\circ$ (Tangent perpendicular to line through centre)

$\angle FCA = 90^\circ$ (Angles in semi-circle)

$\angle OFC + OA'C = 90^\circ$ (Angles in triangle)

therefore

$\angle OFC = \angle CAE$

but

$\angle OFC = \angle ABC$ (Angles in same segment)

therefore

$\angle CAE = \angle ABC$

Given: DE is a tangent to circle centre O at A.

B and C are points on the circle.
Theorems and Worked Examples
Theorem 1

The line drawn through the centre of a circle and bisecting a chord, is perpendicular to the chord.

Converse:
The line drawn through the centre of a circle and perpendicular to a chord, bisects the chord.

\[ \hat{N}_1 = 90^\circ; \text{ radius bisects chord } \]

\[ \hat{P}_1 = 90^\circ; \text{ radius bisects chord } \]

\[ AB = BC; \text{ radius } \perp \text{ chord } \]

\[ AP = PB; \text{ radius } \perp \text{ chord } \]
WORKED EXAMPLE 1
(I DO)

In the diagram, $O$ is the centre of the circle. Chord $AC$ is perpendicular to radius $OD$ at $B$. $OB = 2x$ units and $AC = 8x$ units.

Show that the length of $BD$ is $2x(\sqrt{5} - 1)$ units.
WORKED EXAMPLE 1

SOLUTION

\[ OB = 2x \]
\[ BC = 9x \]
\[ AB = BC = 4x \quad \text{radius \ perpendicular \ chord} \]

In \( \triangle OBC \):
\[ OC^2 = OB^2 + BC^2 \]
\[ \therefore OC^2 = (2x)^2 + (4x)^2 \]
\[ = 4x^2 + 16x^2 \]
\[ = 20x^2 \]
\[ \therefore OC = \sqrt{20x} \]
\[ \therefore OC = \sqrt{4 \times 5}x \]
\[ \therefore OC = 2\sqrt{5}x \]

\[ BD = OD - OB \]

but \( OD = OC \) \quad \text{both radii}.

\[ \therefore BD = 2\sqrt{5}x - 2x \]
\[ \therefore BD = 2x(\sqrt{5} - 1) \]

In the diagram, \( O \) is the centre of the circle. Chord \( AC \) is perpendicular to radius \( OD \) at \( B \). \( OB = 2x \) units and \( AC = 8x \) units.

Show that the length of \( BD \) is \( 2x(\sqrt{5} - 1) \) units.
In the diagram, O is the centre of circle ABD. F is a point on chord AB such that DOF \perp AB. AB = FD = 8\ cm and OF = x\ cm.

Determine the length of the radius of the circle.
**WORKED EXAMPLE 2**

**SOLUTION**

\[
AF = FB = 4 \text{ cm}; \quad \text{radius} \perp \text{ chord}
\]

\[
FD = x + r = 8
\]

\[
\therefore \quad r = 8 - x
\]

In \(\triangle FOB\),
\[
OF^2 + FB^2 = OB^2
\]

\[
x^2 + 4^2 = (8-x)^2
\]

\[
\therefore \quad x^2 + 16 = 64 - 16x + x^2
\]

\[
16x = 48
\]

\[
\therefore \quad x = 3
\]

\[
\therefore \quad \text{radius} = 8 - x = 8 - 3 = 5 \text{ cm}
\]

In the diagram, \(O\) is the centre of circle \(ABD\). \(F\) is a point on chord \(AB\) such that \(DOF \perp AB\). \(AB = FD = 8 \text{ cm}\) and \(OF = x \text{ cm}\).

Determine the length of the radius of the circle.
Theorem 7

Tangents

Tangents and radii

<table>
<thead>
<tr>
<th>Theorem Statement</th>
<th>Acceptable Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>The angle formed by a tangent and a radius of a circle at the point of contact is 90°</td>
<td>$\hat{T}_1 = 90^\circ;$ tangent $\perp$ radius</td>
</tr>
</tbody>
</table>
Theorem 8

PQRO is a cyclic quadrilateral; opposite angles add up to 180°.
WORKED EXAMPLE 3
(I DO)

O is the centre of the circle PTR. N is a point on chord RP such that ON ⊥ PR. RS and PS are tangents to the circle at R and P respectively.

RS = 15 units; TS = 9 units; RPS = 42.83°.

11.1 Calculate the size of NÒR.

11.2 Calculate the length of the radius of the circle.
WORKED EXAMPLE 3

SOLUTION

1a) Draw radius OP

\[ \hat{O}PS = \hat{O}RS = 90^\circ \] tangent \( \perp \) radius

\[ \therefore \] OPSR is a cyclic quad; opp angles are supplementary

In \( \triangle PRS \),

\[ PS = SR \] tangents drawn from S

\[ \hat{P}SR = 42.83^\circ \]

and \( \hat{RSP} = 94.34^\circ \) \( \because \) angles of \( \triangle \)

\[ \therefore \hat{R}OP = 95.66^\circ \]

But

In \( \triangle ORNO \) and \( \triangle PNO \)

ON = ON \( \therefore \) common

OK = OP \( \therefore \) both radii

NR = NP \( \therefore \) radius \( \perp \) chord

\[ \therefore \triangle ORNO \cong \triangle PNO \ (SSS) \]

\[ \therefore \hat{1} = \hat{2} = 42.83^\circ \]
WORKED EXAMPLE 3

SOLUTION

1. In $\triangle ORS$, tangent is $1$. radius

$$OR^2 + RS^2 = OS^2$$

$\therefore r^2 + 15^2 = (9 + r)^2$

$\therefore r^2 + 225 = 81 + 18r + r^2$

$\therefore 18r = 144$

$\therefore r = 8 \text{ cm}$
A circle with a diameter of 220 mm is divided by chord AB into two segments, as shown in the diagram below. The height of one segment is 60 mm.

Calculate the length of chord AB.
WORKED EXAMPLE 4
SOLUTION

In $\triangle AOT$,

$$AO^2 = OT^2 + AT^2$$

$$110^2 = 50^2 + AT^2$$

$$\therefore AT^2 = 9600$$

$$\therefore AT = 98 \text{ mm}.$$  

$$\therefore AB = 196 \text{ mm; radius } 1 \text{ chord}.$$
Theorem 9

TANGENT/CHORD THEOREM

The angle between a tangent and a chord through the point of contact is equal to the angle subtended by that chord in the alternate segment.

(tangent/chord theorem)
In the diagram alongside, $O$ is the centre of the circle and $PTQ$ is a tangent.
Determine the size of $\angle MOT$
WORKED EXAMPLE 5
SOLUTION

In the diagram alongside, O is the centre of the circle and PTQ is a tangent. Determine the size of \( \hat{MOT} \)

\[
\hat{T_1} + 77^\circ = 90^\circ; \text{ rad } \perp \tan \quad \therefore \quad \hat{T_1} = 13^\circ \\
\hat{O} + 13^\circ + 13^\circ = 180^\circ; \text{ angles of } \triangle MOT \quad \therefore \quad \hat{O} = 154^\circ
\]
(I DO)

If a point is not on the circle nor at the centre, then you must use basic triangle geometry or parallel lines.

(c) In the diagram below, A, B and C are points on a circle. CB produced meets the tangent from A at the point T. The bisector of \( \hat{A}TB \) intersects AB and AC at X and Y respectively.

Let \( \hat{TAB} = \beta \) and \( \hat{ATX} = \hat{XIB} = \alpha \).

Prove that \( \triangle AXY \) is isosceles.
(c) In the diagram below, A, B and C are points on a circle. CB produced meets the tangent from A at the point T. The bisector of $\hat{ATB}$ intersects AB and AC at X and Y respectively.

Let $\hat{TAB} = \beta$ and $\hat{ATX} = \hat{XTB} = \alpha$.

Prove that $\triangle AXY$ is isosceles.

\[
\begin{align*}
(c) \quad \hat{AXY} &= \alpha + \beta; \quad \text{ext. angle of } \triangle ATX \\
\hat{C} &= \beta; \quad \text{tan/chord theorem} \quad \text{and} \quad \hat{AYX} &= \alpha + \beta; \quad \text{ext. angle of } \triangle TYC
\end{align*}
\]

Therefore the triangle is isosceles.
In the diagram alongside, we say

- KL subtends an angle at T.
- T is subtended by KL.

(a) In the diagram alongside, show the angle that is subtended by MN.
Theorem 3
ANGLE IN SEMI CIRCLE

Given:
BD is diameter

(a) The *angle in the semi-circle* is the angle subtended by the diameter of a circle at the circumference.

In only one of the above is the angle subtended by the diameter 90°.
In the diagram below, a circle centre O is drawn.

- AB is a diameter of the circle and C is a point on the circle.
- AB produced meets the tangent at C at D.
- AC = DC.

Determine, giving reasons, the size of \( \hat{A} \).
WORKED EXAMPLE 8
SOLUTION

Let $\hat{C}_2 = x$

$\hat{C}_1 = 90^\circ$ (angle in semi-circle)

$\hat{C}_2 = \hat{A} = x$ (tangent/chord theorem)

$\hat{D} = \hat{A} = x$ (isosceles $\triangle$).

$\therefore$ In $\triangle ABD$,

$x + x + 90 + x = 180^\circ$

$\therefore 3x = 90^\circ$

$\therefore x = 30^\circ$

$\therefore \hat{A} = 30^\circ$
AB is a diameter of the circle ABCD. OD is drawn parallel to BC and meets AC in E.

If the radius is 10 cm and AC = 16 cm, calculate the length of ED.
WORKED EXAMPLE 9
(YOU DO)

\[ \hat{C} = 90^\circ; \text{ angle in semi-circle.} \]

\[ \hat{E} = 90^\circ; \text{ corresponding angle OD/BC} \]

\[ \therefore \ AE = EC = 8 \text{cm}; \text{ radius \ perpendicular \ to \ chord}. \]

In \( \triangle AOE \)

\[ AO^2 = AE^2 + OE^2 \]

\[ \therefore \ 10^2 = 8^2 + OE^2 \]

\[ \therefore \ 100 = 64 + OE^2 \]

\[ \therefore \ OE = 6 \]

\[ \therefore \ ED = 10 - 6 = 4 \text{cm} \ (\text{OD is a radius}) \]
ARCS AND CHORDS

In the diagram alongside, we say

- arc KL subtends an angle at T.
- T is subtended by arc KL.

(a) In the diagrams below, P, Q and R are three points on the circle. Illustrate the statement given below each circle.

Arc PR subtends an angle at Q
Arc QR subtends an angle at P
Theorem 2
ANGLE AT CENTRE

\[ \begin{align*}
\text{angle at centre } x &= 42°; \\
\text{angle at centre } c &= 22.5°; \\
\text{angle at centre } e &= 70°; \\
\text{angle at centre } d &= 200°;
\end{align*} \]
WORKED EXAMPLE 10
(YOU DO)

\[ \hat{G} = x \quad ; \quad \text{angle at centre} \]
\[ \overline{H}_1 = x \quad ; \quad \text{alternate angles} \quad \text{GJ} \parallel \text{KH} \]
\[ \widehat{f}_1 + \widehat{f}_2 = x \quad ; \quad \text{tan/chord theorem} \]

\[ \overline{H}_1 + \overline{H}_2 = 90^\circ \quad ; \quad \text{tan} \perp \text{radius} \]
\[ \therefore \overline{H}_2 = 90^\circ - x \]

\[ \overline{H}_3 + \widehat{f}_1 = 180^\circ - 2x \quad ; \quad \text{angles of a } \Delta \]
But \( OJ = OH \); both radii
\[ \therefore \widehat{f}_1 + \overline{H}_3 = 90^\circ - x \]

8.1.1 Name, giving reasons, THREE angles, each equal to \( x \).
8.1.2 Prove that \( \overline{H}_2 = \overline{H}_3 \).
In the diagram, O is the centre of the circle. ST is a tangent to the circle at T. M and P are points on the circle such that TM = MP. OT, OP and TP are drawn. Let $\hat{O}_1 = x$.

Prove, with reasons, that $\hat{STM} = \frac{1}{4}x$. 
\[ \hat{\theta}_2 = 360 - \alpha \] angles around a point

\[ \hat{\theta}_2 + \hat{\rho}_1 = 180 - (180 - \frac{\alpha}{2}) \] angles at centre

\[ \hat{\theta}_2 + \hat{\rho}_1 = \frac{\alpha}{2} \]

but \( TM = MP \)

\[ \hat{\theta}_2 = \hat{\rho}_1 = \frac{\alpha}{4} \]

\[ \hat{\theta}_1 = \hat{\rho}_1 = \frac{\alpha}{4} \]

\( \tan/\text{chord} \)
WORKED EXAMPLE 12
(YOU DO)

In the diagram, S is the centre of circle PQRT. PT is a diameter. \(\hat{RST} = x - 8^\circ\) and \(\hat{PQR} = 2x - 40^\circ\).

Determine the value of \(x\).
In the diagram, S is the centre of circle PQRT. PT is a diameter. 
$
\hat{RST} = x - 8^\circ \text{ and } \hat{PQR} = 2x - 40^\circ.
$

Determine the value of $x$. 

**Reflex $\hat{S} = 180 + x - 8^\circ = 172^\circ + x$.**

\[
\hat{Q} = \frac{1}{2} (\text{Reflex } \hat{S}) \text{; angle at centre}
\]

\[
2x - 40^\circ = \frac{1}{2} (172^\circ + x)
\]

\[
2x - 40^\circ = 86 + \frac{1}{2} x
\]

\[
\frac{3}{2} x = 126
\]

\[
x = 84^\circ
\]
Converse of Theorem 3
If a line subtends an angle of 90° at a point then the line is a diameter of the circle passing through the point.

(Converse of angle in semi-circle)

If then

A point is the centre of the circle if it is equidistant from three other points on the circle.

If \( AE = EC = BE \) then \( E \) is the centre of the circle.

The following diagrams illustrate that \( BE = EC \) but \( E \) is not the centre of the circle. We need all three line segments to be equal.
(c) In the diagram below, triangle ABC is an equilateral triangle. EC bisects \( \angle ACB \) and EA bisects \( \angle CAB \).

AE = BE.

1. Explain why AC is not a diameter of the circle passing through A, E and C.
2. Prove that E is the centre of the circle passing through A, B and C.
WORKED EXAMPLE 13

SOLUTION

(c) (1) \( \hat{A} = \hat{B} = \hat{C} = 60^\circ \); equilateral \( \Delta \)

\( \hat{A}_1 = \hat{A}_2 = 30^\circ \); given \( \hat{C}_1 = \hat{C}_2 = 30^\circ \); given

\( \hat{E}_1 + \hat{A}_1 + \hat{C}_1 = 180^\circ \); angles of \( \Delta \)AEC \( \therefore \) \( \hat{E}_1 = 120^\circ \)

AC subtends at E \( \therefore \) Therefore the converse of angle in semi circle does not hold.

Therefore AC is not a diameter.

(2) \( \hat{A}_1 = \hat{C}_1 = 30^\circ \); proven in (a) \( \quad \) AE = CE; isos. \( \Delta \)ACE

But AE = EB; given \( \quad \therefore \) Therefore, AE = CE = EB

Therefore E is the centre of the circle; equidistant from 3 points on the circle.
(b) In the diagram alongside, $O$ is the centre of the semi-circle. $OD \parallel BC$ and $\hat{BEC} = 36^\circ$.

(1) Prove that $OA$ is the diameter of the circle passing through $AO$ and $E$. 

\[\text{Diagram with angles and lines described} \]
WORKED EXAMPLE 14
SOLUTION
(YOU DO)

(b) In the diagram alongside, O is the centre of the semi-circle. OD//BC and $\hat{BEC} = 36^\circ$.

(1) Prove that OA is the diameter of the circle

(2) Determine the size of $\hat{D}$

(b) (1) $\hat{C}_{2} = 90^\circ$; angle in semi circle. $\therefore \hat{B} = 54^\circ$; angles of $\triangle ABC$

$\hat{O}_{1} = 54^\circ$; corres. Angles OD//BC $\therefore \hat{E}_{1} = 90^\circ$; angles of $\triangle AOE$

Therefore, OA is diameter; converse of angle in semi circle.
The term "angles in same segment" refers to angles that are subtended by the same chord at the circumference and also in the same segment.
Theorem 4

Angles in same segment are equal.

*(Angles in same segment)*

\[ \hat{A} = \hat{B} \text{ and } \angle APB = \angle AQB \]
WORKED EXAMPLE 15
(I DO)

In the diagram alongside, O is the centre of the circle.

(1) Is \( \hat{B}_1 + \hat{B}_2 = 90^\circ \)? Explain

(2) Why is \( \hat{A}_2 = \hat{B}_2 \)? Explain

(3) Which two angles are equal and subtended by Chord AB?

In the diagram alongside, A, B, C and D are four points

(1) Determine the size of \( \hat{B} \)?

(2) Which angle is equal to \( \hat{C} \)?
WORKED EXAMPLE 16
(YOU DO)

- B, C, D, E and F lie on the circle centre O.
- Lines AB and AF are tangents to the circle at B and F respectively.
- Line BE passes through O.

(a) Prove that $\hat{C} + \hat{D} = 90^\circ$.

(b) If $\hat{D} = 38^\circ$, determine the size of $\angle BAF$. 
WORKED EXAMPLE 16

SOLUTION

\[ \hat{C}_1 + \hat{C}_2 = 90^\circ \quad \text{angle in semi-circle} \]

but \[ \hat{C}_2 = \hat{D} \quad \text{angles in same segment} \]

\[ \therefore \hat{C}_1 + \hat{D} = 90^\circ \]

Join \( F \) to \( O \)

\[ \hat{A}BO = 90^\circ \quad \text{tangent \perp \text{rad}} \]

\[ \hat{F}OB = 90^\circ \quad \text{tangent \perp \text{rad}} \]

\[ \therefore \text{ABOF is a cyclic quad}; \text{opp angles are suppl.} \]

\[ \therefore A = \hat{F}OE \quad \text{ext \ < \ of \ \angle} \]

but \[ \hat{F}OE = 2\hat{D} = 76^\circ \quad \text{angle at centre} \]

\[ \therefore A = 76^\circ \]

- \( B, C, D, E \) and \( F \) lie on the circle centre \( O \).
- Lines \( AB \) and \( AF \) are tangents to the circle at \( B \) and \( F \) respectively.
- Line \( BE \) passes through \( O \).

(a) Prove that \( \hat{C}_1 + \hat{D} = 90^\circ \).

(b) If \( \hat{D} = 38^\circ \), determine the size of \( \hat{BAF} \).
Important Corollaries

(a) Equal chords subtend equal angles in the same segment. 
(angles in same segment, equal chords)

If \( PQ = AB \) then \( \hat{D} = \hat{C} \)

(b) EQUAL circles are circles that have the same radius.

Equal chords of EQUAL circles subtend equal angles in the same segment. 
(Equal Circles, Equal Chords, angles in same segment)

If \( MN = BC \) then \( \hat{A} = \hat{R} \)
O is the centre of circle TNSPR. \( \overset{\frown}{P\overset{\frown}{S}} = 60^\circ \) and \( PS = NT \).

Calculate the size of:

- 9.2.1 \( \overset{\frown}{P\overset{\frown}{S}} \)
- 9.2.2 \( \overset{\frown}{N\overset{\frown}{T}} \)
WORKED EXAMPLE 18

(YOU DO)

In the figure, QS and PR are diameters of the circle with centre O such that PQ // SR. PS is produced to T. N is a point on the circle such that \( \hat{\text{O}}_1 = \hat{\text{O}}_2 \).

SN is drawn.

RS intersects QN at M. \( \hat{\text{S}}_1 = 48^\circ \)

10.2.1 Determine, with reasons, the size of:

(a) \( \hat{\text{O}}_1 \) \( \hspace{1cm} (3) \)

(b) \( \hat{\text{R}} \) \( \hspace{1cm} (2) \)

(c) \( \hat{\text{M}}_1 \) \( \hspace{1cm} (2) \)

10.2.2 Prove that ST is a tangent to the circle passing through M, N and S. \( \hspace{1cm} [14] \)
WORKED EXAMPLE 19
(I DO)

(i) In the diagram below, two EQUAL circles are drawn.
    O is the centre of the one circle.
    AB = AC and BC is a common chord of the circles.
    Determine \( \hat{OCA} \)

\[ \hat{A} = \hat{D} = 30^\circ; \text{ equal circles; equal chords; angles in same segment} \]
\[ \hat{CBA} + \hat{BAC} + \hat{A} = 180^\circ; \text{ angles of } \triangle ABC \]
\[ \text{Therefore } \hat{B} + \hat{C} = 150^\circ \]

But \( \hat{CBA} = \hat{BAC} \); isos. \( \triangle ABC \)
\[ \text{Therefore } \hat{CBA} = \hat{BAC} = 75^\circ \]

\[ \therefore \hat{O} = 150^\circ; \text{ angle at centre} \]
\[ \text{Also, } \hat{OCA} + \hat{OAC} + \hat{O} = 180^\circ; \text{ angles of } \triangle OAC \]

\[ \text{Therefore } \hat{OCA} + \hat{OAC} = 30^\circ. \text{ Also, } \hat{OCA} = \hat{OAC}; \text{ isos. } \triangle OCA \]

\[ \text{Therefore } \hat{OCA} = 15^\circ \]
In the diagram below:
- Two major segments of circles are drawn with BD a common chord.
- Both circles have equal diameters.
- Chord AB is parallel to chord CD.

Prove that ABCD is a parallelogram.
Practice Makes Perfect
PRACTICE EXAMPLE 1

In the diagram below, A, B and C are points on the circle centre O.
AD = DB.
Prove that ACDO is a cyclic quadrilateral.
(h) In the diagram below, A, B and C are points on the circle centre O. 
AD = DB.
Prove that ACDO is a cyclic quadrilateral.

\[
\begin{align*}
\hat{AOC} &= 2\hat{B} = 2x; \text{ angle at centre} \\
\hat{DAB} &= x; \text{ isos. } \Delta \\
\therefore \quad \hat{CDA} &= \hat{DAB} + \hat{B} = 2x; \text{ ext. angle of } \Delta DAB \\
\text{Therefore, ACDO is a cyclic quad; converse of angles in same segment.}
\end{align*}
\]
In the diagram below, O is the centre of the semi-circle ABCD.
AC and BD intersect at E.
AD//OC

Determine the size of $\angle AED$
(1) In the diagram below, O is the centre of the semi-circle ABCD.

AC and BD intersect at E.

AD // OC

Determine the size of $\hat{AED}$

(1) Draw DO  

$\hat{AED} = \hat{EAB} + 24^\circ$; ext. angle of $\triangle D = 90^\circ$; angle in semi circle.

$\hat{OFB} = 90^\circ$; corres. angles AD // OC  

$\hat{CDB} = 66^\circ$; angles of a $\triangle$

$\hat{EAB} = 33^\circ$; angle at centre  
Therefore $\hat{AED} = 33^\circ + 24^\circ = 57^\circ$
(j) In the diagram, O is the centre of the circle.
RQ is a tangent to the circle.
TQ = TR.
Let $\hat{O} = 2x$
Prove that PQ = QR
(j) In the diagram, O is the centre of the circle.  
RQ is a tangent to the circle.  
TQ = TR.  

Prove that PQ = QR  

\( \hat{T}_1 = \frac{1}{2} \hat{O} = x \); angle at centre.  
\( \hat{T}_1 = \hat{R} + \hat{Q}_3 \); ext. angle of \( \triangle TQR \)  
\( \hat{R} = \hat{Q}_3 \); isos. \( \triangle TRQ \)  
\( \therefore \hat{R} = \frac{x}{2} \).  
\( \hat{P}_2 = \hat{Q}_3 = \frac{x}{2} \); tan/chord theorem.  

Therefore \( \hat{P}_2 = \hat{R} \).  
Therefore PQ = QR
(i) (1) What deduction can be made from both statements if \( \hat{A} = \hat{J} \) and \( \hat{W} = \hat{J} \)

(2) In the diagram below, PQ is a chord of a circle centre M. PQ∥AB.

Prove that ABSR is a cyclic quadrilateral.
(2) In the diagram below, PQ is a chord of a circle centre M. PQ // AB.

Prove that ABSR is a cyclic quadrilateral.

(2) Draw line RS. \(\hat{Q} = \hat{PSR}\); angles in same segment. \(\hat{Q} = \hat{A}\); alt angles PQ // AB
Therefore \(\hat{A} = \hat{PSR}\). Therefore ABSR is a cyclic quad.; converse of ext. angle of cyclic quad.
(a) In the diagram below, A, B, C and D are points on a circle.

\[ \hat{CDB} = 36^\circ, \quad BD \text{ is a diameter and } AB = AC. \]

Determine the size of \( \hat{ABD} \)
(a) In the diagram below, A, B, C and D are points on a circle.

\( \hat{CDB} = 36^\circ \), BD is a diameter and \( AB = AC \).

Determine the size of \( \hat{ABD} \)

\[
\begin{align*}
\hat{A} &= \hat{D} = 36^\circ; \text{ angles in same segment} \\
\therefore \hat{C_2} &= \hat{B} = 72^\circ; \text{ isos } \triangle \\
\hat{C_1} + \hat{C_2} &= 90^\circ; \text{ angle in semi circle} \quad \therefore \hat{C_1} = 18^\circ \\
\therefore \hat{B_1} &= 18^\circ; \text{ angles in same segment}
\end{align*}
\]
(e) In the diagram below, two intersecting circles are given.

- CA is the diameter of the smaller circle.
- The two circles intersect at A and B.

1. Prove that $\hat{CPQ} = 90^\circ$
2. Prove that CBQ is a straight line.
(e) In the diagram below, two intersecting circles are given.
- CA is the diameter of the smaller circle.
- The two circles intersect at A and B.

(1) Prove that $\hat{CPQ} = 90^\circ$
(2) Prove that CBQ is a str. line.
(d) In the diagram below, O is the centre of a semicircle ABCD. 

AB$\parallel$OC and $\hat{BAD} = 38^\circ$.

Determine the size of $\hat{BDC}$
(d) In the diagram below, O is the centre of a semicircle ABCD.

AB//OC and \(\hat{BAD} = 38^\circ\).

Determine the size of \(\hat{BDC}\)

\[
\begin{align*}
\hat{BOC} &= 38^\circ; \text{ alt. angles } AB//OC \\
\hat{BDC} &= 19^\circ
\end{align*}
\]
(l) In the diagram below, AB and BC are chords of the circle. D is a point on the circle so that AD is perpendicular to BC. DE // BC

Prove that \( \hat{EAC} + \hat{ABC} = 90^\circ \)
(i) In the diagram below, $AB$ and $BC$ are chords of the circle.

$D$ is a point on the circle so that $AD$ is perpendicular to $BC$.

$DE//BC$

Prove that $\angle E\hat{A}C + \angle A\hat{B}C = 90^\circ$

(m) Join $AE$, $AC$ and $DC$.

$\angle E\hat{A}C = \angle CDE$ and $\angle A\hat{B}C = \angle A\hat{D}C$; angles in same segment

Therefore $\angle E\hat{A}C + \angle A\hat{B}C = \angle CDE + \angle A\hat{D}C$

But $\angle CDE + \angle A\hat{D}C = 90^\circ$; co-int. angles $BC//ED$

Therefore $\angle E\hat{A}C + \angle A\hat{B}C = 90^\circ$
(h) *Concentric circles are circles with the same centre*

In the diagram below O is the centre of the circle.
CD is a chord of the larger circle and a secant of the smaller circle.
CD cuts the smaller circle in P and B.
CD = 20, BP = 8 and OT = 3

Determine the width of the shaded ring.
(h) *Concentric circles are circles with the same centre*

In the diagram below O is the centre of the circle. CD is a chord of the larger circle and a secant of the smaller circle. CD cuts the smaller circle in P and B. CD = 20, BP = 8 and OT = 3

Determine the width of the shaded ring.

(h) The width of the ring is the difference in the radii.

\[ BT = TP, \ OT \perp BP \quad \text{and} \quad DT = CT, \ OT \perp CD \]

In \( \triangle OTD \),

\[ OD^2 = OT^2 + DT^2 \quad \therefore \quad OD^2 = 3^2 + 10^2 = 109 \quad \therefore \quad OD = \sqrt{109} \]

In \( \triangle OTP \),

\[ OP^2 = OT^2 + TP^2 \quad \therefore \quad OP^2 = 3^2 + 4^2 = 25 \quad \therefore \quad OP = 5 \]

Therefore, width of ring is \( \sqrt{109} - 25 \)
(f) In the diagram below, O is the centre of the circle.

AM = MB = 3
MS = 1

Determine the length of EB
(f) In the diagram below, O is the centre of the circle.

\[ AM = MB = 3 \]
\[ MS = 1 \]

Determine the length of EB

(e) \( \hat{M}_1 = 90^\circ ; AM = MB \)

In \( \triangle OMB, \)  \[ OM^2 = MB^2 + OB^2 \]  \[ \therefore r^2 = (r - 1)^2 + 3^2 \]
\[ r^2 = r^2 - 2r + 1 + 9 \]
\[ \therefore 2r = 10 \]
\[ \therefore r = 5 \]

In \( \triangle EMB, \)  \[ EB^2 = MB^2 + EM^2 \]  \[ \therefore EB^2 = 3^2 + 9^2 = 90 \]
\[ \therefore EB = \sqrt{90} \]