MATHEMATICS

Webinar 1: Sunday 19th April 15h00-17h00

Functions and Graphs: (Quadratic)

Excludes Exponential, Hyperbola, Inverse functions, logarithmic functions (Future Session)

Approximately 35 out 150- Paper 1

CONTENTS

GENERAL (ALL GRAPHS)

We use graphs to

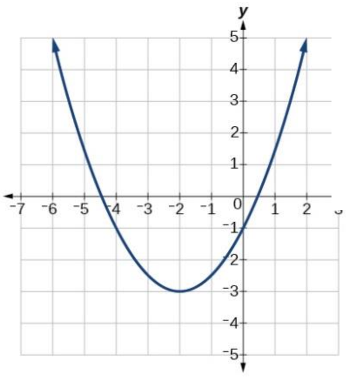
(1) Determine point(s) of Intersection (Task 2)
(2) Solve Equations (Task 2)
(3) Solve Inequalities (Task 2)
(4) Solve Nature of Roots Problems (Task 3)
(5) Solve Length and Area Problems(Task 4)
(6) Solve Problems in Real World Contexts (Task 3)

QUADRATIC FUNCTION

1. Sketch the graph of any Parabola of the form $y = ax^2 + bx + c$ or $y = a(x-p)^2 + q$ and understand
   a. domain, range, minimum, maximum
   b. Determine the axis of symmetry (turning point) by
      i. First writing the expression in the form $y = a(x-p)^2 + q$
      ii. Using the midpoint of two x-intercepts or any other two symmetrical points
      iii. Using $x = \frac{-b}{2a}$
      iv. Using the derivative(Calculus)- Will deal with this when we revise Calculus.
   c. Intercepts with axes.
   d. the effect on the graph if values of *a*, *b*, c, *p* and *q* change in the equations $y = ax^2 + bx + c$ and $y = a(x-p)^2 + q$
2. Determine the equation of a parabola given anyone of the following scenarios:
   a. Given turning point
   b. Given x-intercepts
   c. Given arbitrary points on the parabola

QUADRATIC FUNCTION

(a) Sketch the graph of any Parabola of the form $y = ax^2 + bx + c$ or $y = a(x-p)^2 + q$ and understand
   a. domain, range, minimum, maximum



Domain=$(-\infty;\infty)$

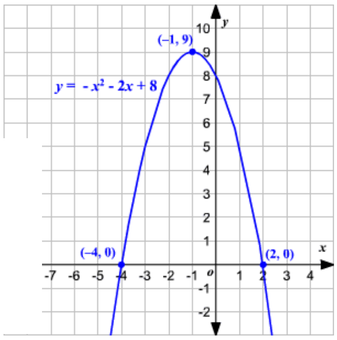
Range= $[-3;\infty)$

Minimum=-3

Domain=$(-\infty;\infty)$

Range= $(-\infty;9]$

Maximum = 9



b. Determine the axis of symmetry (turning point) by

i. First writing the expression in the form $y = a(x-p)^2 + q$

Method 1: Preferred

$$x^2 + 3x - 5 = (x + \frac{3}{2})^2 - \frac{9}{4} - 5$$

$$= (x + \frac{3}{2})^2 - \frac{9}{4} - \frac{20}{4}$$

$$= (x + \frac{3}{2})^2 - \frac{29}{4}$$

Tpt$\left(-\frac{3}{2}; -\frac{29}{4}\right)$

Example: Express $2x^2 - 8x - 13$ in the form $a(x-p)^2 + q$

$$A = 2x^2 - 8x - 13$$

$$\frac{A}{2} = x^2 - 4x - \frac{13}{2}$$

$$\frac{A}{2} = (x-2)^2 - 4 - \frac{13}{2}$$

$$\frac{A}{2} = (x-2)^2 - \frac{21}{2}$$

$$A = 2(x-2)^2 - 21$$

Method 2: (Optional)

Express $2x^2 - 6x + 7$ in the form $p(x + q)^2 + r$

$2x^2 - 6x + 7 = p(x + q)^2 + r$

Obviously $p = 2$ to obtain $2x^2$

$2x^2 - 6x + 7 = 2(x + q)^2 + r$

$= 2(x + q)(x + q) + r$

$= 2(x^2 + 2qx + q^2) + r$

$= 2x^2 + 4qx + 2q^2 + r$

So $2x^2 - 6x + 7 = 2x^2 + 4qx + 2q^2 + r$

$$\therefore 4q = -6 \quad \therefore q = -\frac{3}{2}$$

$$2q^2 + r = 7 \quad \therefore 2\left(-\frac{3}{2}\right)^2 + r = 7 \quad \therefore r = \frac{5}{2}$$

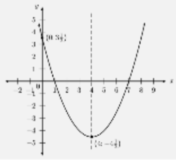
$$2x^2 - 6x + 7 = 2\left(x - \frac{3}{2}\right)^2 + \frac{5}{2}$$

TASK 1

Question

1

Write down the domain of the function alongside
(1 Point)



Enter your math answer

Correct answers: $(-\infty; \infty)$

2

Write down the Range of the function above
(1 Point)

Enter your math answer

Correct answers: $[-4,5; \infty)$

3

Which of the following is true
(1 Point)

- ○ The function above has a minimum value of -4,5 ✓
- ○ The function above has a maximum of -4,5
- ○ The function above has a minimum value of 4
- ○ The function above has a maximum value of 4

4

Express f(x) in the form of a(x-p)^2+q and hence write down the co-ordinates of the turning points.
(3 Points)

$f(x) = 4x^2 - 8x + 12$

Enter your answer

Solution

TASK 1

① $\mathbb{R}$ or $(-\infty; \infty)$

② $[-4\tfrac{1}{2}; \infty)$

③ Min of $-4,5$

④ $f(x) = 4x^2 - 8x + 12$

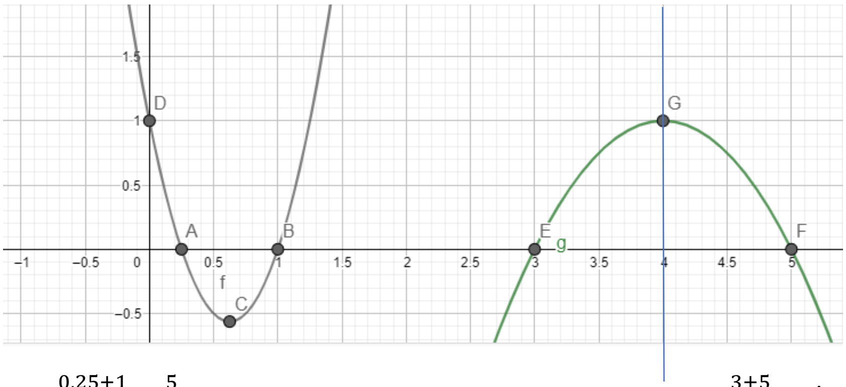
$$\frac{f(x)}{4} = x^2 - 2x + 3$$

$$\frac{f(x)}{4} = (x-1)^2 - 1 + 3$$

$$\frac{f(x)}{4} = (x-1)^2 + 2$$

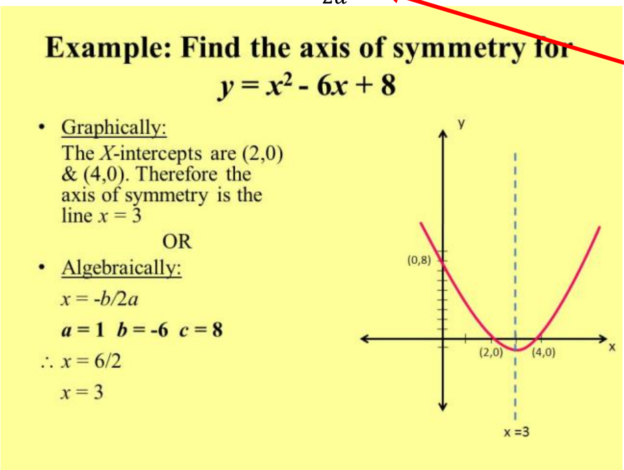
$$f(x) = 4(x-1)^2 + 8$$

ii. Using the midpoint of two x-intercepts or any other two symmetrical points



$x = \frac{0,25+1}{2} = \frac{5}{8}$ $\qquad$ $x = \frac{3+5}{2} = 4$

iii. Using $x = \frac{-b}{2a}$



$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$x_{Tpt} = \frac{sum}{2} = \frac{\frac{-b-b}{2a}}{2} = -\frac{b}{2a}$$

iv. Using the derivative(Calculus)- Will deal with this when we revise Calculus.

c. Intercepts with axes.

x-intercepts (let y = 0, then solve for x)	y-intercepts (let x = 0, then solve for y)
$y = x^2 - 2x - 3$ $0 = x^2 - 2x - 3$ $0 = (x+1)(x-3)$ $x_1 = -1\ ,\ x_2 = 3$ ✓	$y = x^2 - 2x - 3$ $y = (0)^2 - 2(0) - 3$ $= 0 - 0 - 3$ $y = -3$ ✓
as points: $(-1,0)$ *and* $(3,0)$	as point: $(0,-3)$

d. the effect on the graph if values of a, b, c, p and q in the equations $y = ax^2 + bx + c$ and $y = a(x-p)^2 + q$

Summary: Effect of a(Arms and Turning Point)

(1) Arms

If ***a*** increases in magnitude, then the arms move closer/tighter.

(2) Turning Point

$$x = -\frac{b}{2a} \quad \text{and} \quad y = ax^2 + bx + c$$

If ***a*** increases (positive or negative), the turning point shifts horizontally left and vertically up.

If ***a*** decreases (positive or negative), then the turning point shifts horizontally right and vertically down.

Use Geogebra to show above

Summary: Effect of *b*(Turning Point)

$$x = -\frac{b}{2a}$$

If ***a*** is positive and b increases then x_{tpt} decreases. (Horizontal left shift)

If ***a*** is negative and b increases then x_{tpt} increases.(Horizontal right shift)

$$y = c - \frac{b^2}{4a}$$

If ***a*** is positive and b increases in magnitude y_{tpt} decreases. (Vertical shift down)

If ***a*** is negative and b increases in magnitude then y_{tpt} increases.(Vertical shift Up)

$$y = ax^2 + bx + c$$

$$\frac{y}{a} = x^2 + \frac{b}{a}x + \frac{c}{a}$$

$$\frac{y}{a} = \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}$$

$$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

Summary: Effect of changing *c* (y-intercept)

If ***c*** increases the graph shifts vertically up.

If ***c*** decreases the graph shifts vertically down.

Summary: Effect of changing *p* (horizontal shift) $y = a(x-p)^2 + q$

If ***p*** increases the graph shifts horizontally to the right.

Summary: Effect of changing *q* (vertical shift) $y = a(x-p)^2 + q$

If ***q*** increases the graph shifts vertically up.

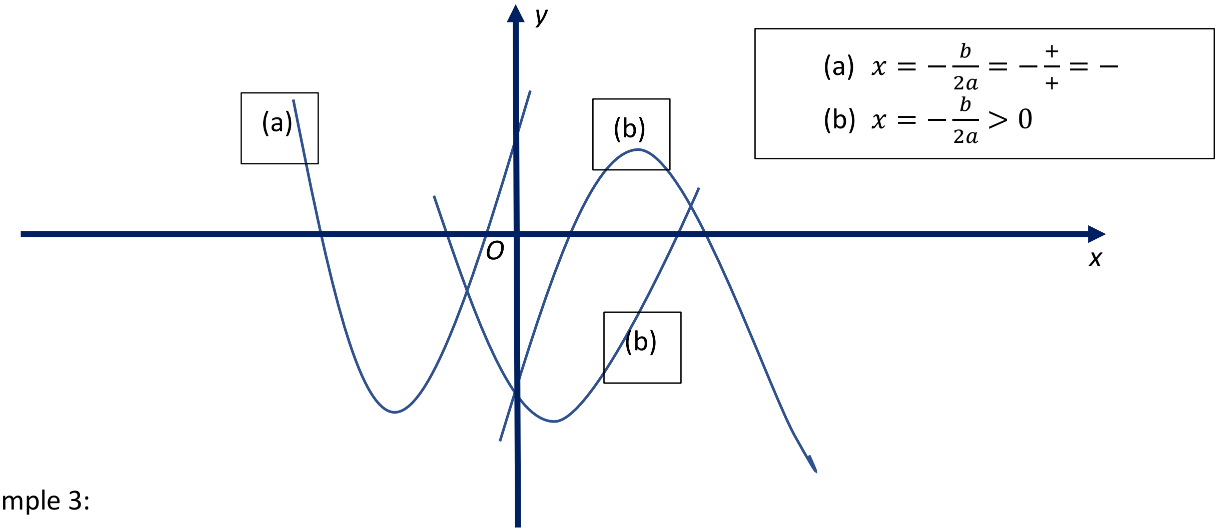
Example 2:

Given: $y = ax^2 + bx + c$

Sketch separate graphs for each of the conditions below:

(a) $a > 0, b > 0 \ and \ c > 0$

(b) $\frac{b}{2a} < 0 \quad and \ c < 0$(The answer below shows two possibilities)



Example 3:

Given: $f(x) = 3 - 2x - x^2$ and $g(x) = x^2 - 1$

(a) Sketch both graphs on the same set of axes. Label clearly all intercepts with axes, turning points and axes of symmetry.

(a) **f(x)**

Tpt: $x = -\frac{b}{2a}$

$x = \frac{-(-2)}{2(-1)}$

$x = -1$

$y = 3 - 2(-1) - (-1)^2$

$y = 3 + 2 - 1$

$y = 4$

Tpt $(-1;4)$

x-ints: let $y = 0$

$3 - 2x - x^2 = 0$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3$ or $x = 1$

g(x)

Tpt: $y = (x-0)^2 - 1$

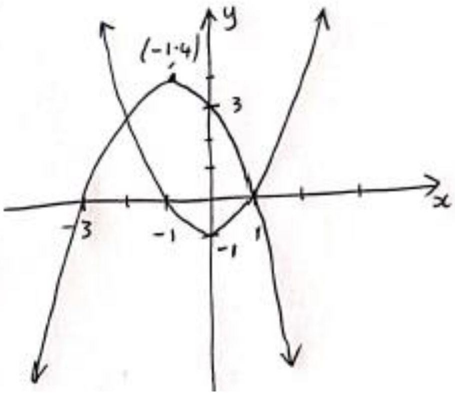
Tpt $(0;-1)$

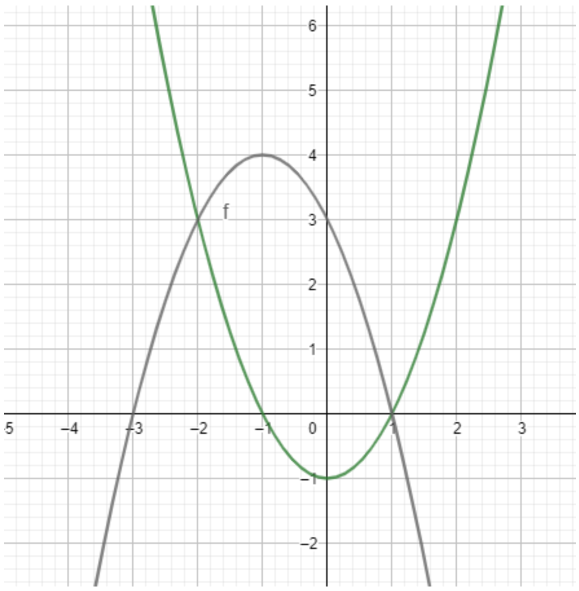
x-ints: let $y = 0$

$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$





(b) Determine the co-ordinates of the point(s) of intersection of the two graphs.

$3 - 2x - x^2 = x^2 - 1$

$\therefore 2x^2 + 2x - 4 = 0$

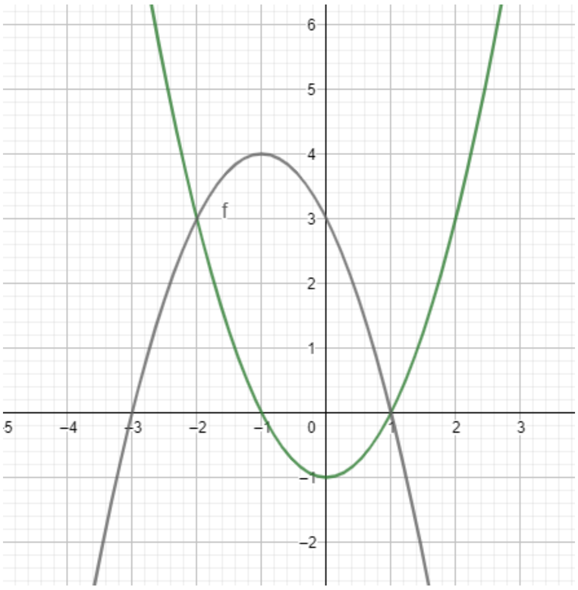
$\therefore x^2 + x - 2 = 0$

$\therefore (x - 1)(x + 2) = 0$

$\therefore x = 1$ or $x = -2$

$y = 0$ $\quad$ $y = 3$

$(1;0)$ $\quad$ $(-2;3)$



(c) Use the graphs to solve the following:

(1) $f(x) > 0$; y>0 ; -3<x<1

(2) $y \leq 0$; -1<x<1

(3) $f(x) \times g(x) \geq 0$; -3<x<-1

(4) $f(x) < g(x)$ $y_f < y_g$ x<-2 or x>1

Task 2:

Question

Task 2 (21 Points)

1

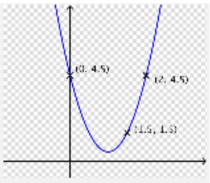
Write down the axis of symmetry of f(x) as well as the co-ordinates of the turning point
(3 Points)

$f(x) = -2(x+3)^2 - 5$

Enter your math answer

Correct answers: $x = -3$ $(-3; -5)$

2



Write down the axis of symmetry for the function in the diagram alongside.
(1 Point)

Enter your math answer

Correct answers: $x = 1$

3

$$y = 2x^2 + 4x + 5$$

Determine the co-ordinates of the turning point of the function given alongside
(3 Points)

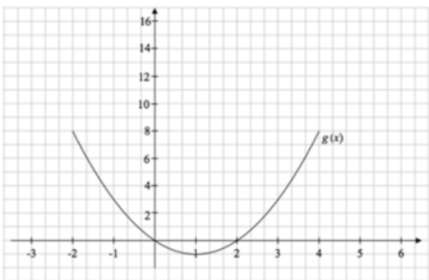
Enter your answer

Correct answers: (-1;3)

4

Copy the graph on a piece of paper and then answer the question. Upload your answer by taking a photo.(4 Points)

In the diagram below, the graph of $g(x)$ is sketched.



On the same diagram, sketch the graph of each of the functions:

(1) $h(x) = g(x) + 2$ (2)

(2) $j(x) = g(x+2)$ (2)

Solution

TASK 2

① $x = -3$ Tpt $(-3; -5)$

② $x = \frac{0+2}{2} = 1$

③ $y = 2x^2+4x+5$

$x = \frac{-b}{2a}$

$x = \frac{-4}{2(2)} = -1$

$\therefore y = 2(-1)^2+4(-1)+5$

$y = 2-4+5$

$y = 3$

$(-1;3)$

$\frac{y}{2} = x^2+2x+\frac{5}{2}$

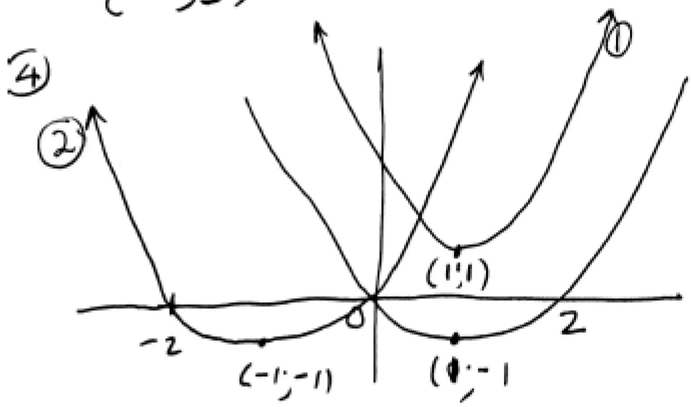
$\frac{y}{2} = (x+1)^2-1+\frac{5}{2}$

$\frac{y}{2} = (x+1)^2+\frac{3}{2}$

$y = 2(x+1)^2+3$

Tpt $(-1;3)$

④

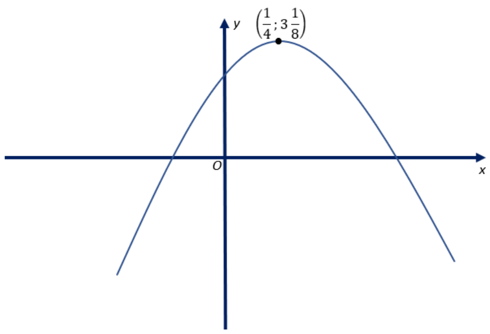


Determining the equation of a parabola

2. Determine the equation of a parabola given anyone of the following scenarios:
   a. Given turning point

Example 4:

The graph of $y = ax^2 + bx + 3$ is given alongside. Determine *a* and *b*.



$$y = a(x-p)^2 + q$$

$$y = a\left(x - \frac{1}{4}\right)^2 + \frac{25}{8}$$

Subs(0;3)

$$3 = a\left(0 - \frac{1}{4}\right)^2 + \frac{25}{8}$$

$$3 = \frac{1}{16}a + \frac{25}{8}$$

$$a = -2$$

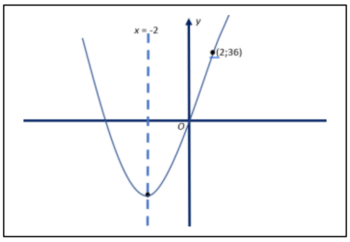
$$y = -2x^2 + bx + 3$$

b. Given x-intercepts

Example 5:

The graph of $f(x) = a(x+p)^2 + q$ is sketched alongside:

Determine the values of *a*, *p* and *q*



$$y = a(x+4)(x-0)$$

Subs(2;36)

$$36 = a(2+4)(2-0)$$

a=3

$$y = 3(x+4)(x-0)$$

$$y = 3(x^2 - 4x) = 3x^2 - 12x$$

$$\frac{y}{3} = x^2 - 4x = (x-2)^2 - 4$$

$$y = 3(x-2)^2 - 12$$

c. Given arbitrary points on the parabola

Example6:

Giben: $y = x^2 + ax + b$

The points (1;-8) and (-2;7) lie on the curve.

Determine the coordinates of the turning point of the curve.

$$y = x^2 + ax + b$$

$$-8 = 1^2 + a + b \ldots\ldots a + b = -9$$

$$7 = (-2)^2 - 2a + b \ldots\ldots\ldots - 2a + b = 3$$

Top – Bottom: 3a=-12 a=-4 -4+b=-9 b=-5

$$y = x^2 - 4x - 5$$

$x = -\frac{b}{2a} = -\frac{-4}{2(1)} = -2$

$$y = (-2)^2 - 4(-2) - 5 = 7$$

Tpt(-2;7)

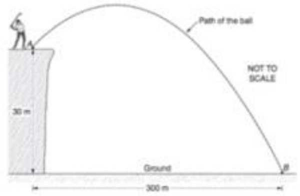
Task 3:

Task 3 (8 Points)

1

Answer the question and then upload by taking a photo.(8 Points)

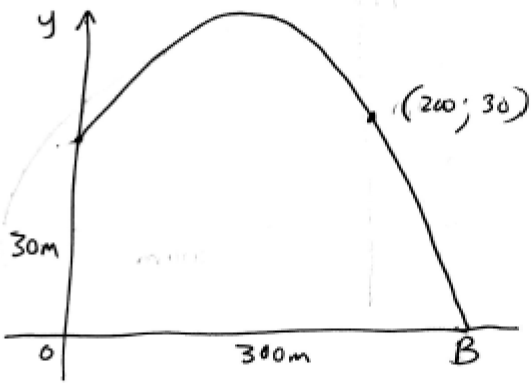
(a) A golf ball is hit from point A to point B.
Point A is 30 metres vertically above the ground.
Point B is 300 metres away from A on the horizontal ground.
The path of the flight of the golf ball is placed in the Cartesian plane so that A is on the y axis and B is on the x axis.
The path of the golf ball is modelled by $y = ax^2 + bx + c$.
The ball passes through the point (200;30).



Determine the maximum height the ball reaches above the ground. (8)

Solution

TASK 3



$\therefore$ Axis of symmetry is $x = 100$.

$\therefore$ x-int is $x = 300$ and $x = -100$

$\therefore y = a(x-300)(x+100)$

Subs (200; 30)

$30 = a(200-300)(200+100)$

$\therefore a = -\frac{1}{100}$

$\therefore y = -\frac{1}{100}(x^2 - 200x - 3000)$

$\therefore y = -\frac{1}{100}x^2 + 2x + 30$

$\therefore$ Max height $= -\frac{1}{100}(100)^2 + 200 + 30 = 130$m