

## ADVANCED PROGRAMME MATHEMATICS

### A. MEANS OF ASSESSMENT

Paper 1 (Core)	2 hours	[200]
Paper 2 (Elective)	1 hour	[100]

**300 marks**

The certification process for Advanced Programme Mathematics consists of one 3-hour external examination which is set by the Independent Examinations Board and quality assured by Umalusi.

### B. REQUIREMENTS

To be read in conjunction with the: *Curriculum Statement Grades 10 – 12 (General) Advanced Programme Mathematics (2006)*.

The Learning Outcomes of the Advanced Programme Mathematics National Curriculum Statement are divided into Core Learning Outcomes (LO 1: Calculus & LO 2 Algebra) and Elective Learning Outcomes (LO 3: Statistics, LO 4: Mathematical modelling & LO 5: Matrices and Graph Theory). Learners will be examined on the Core Learning Outcomes (LO 1 & LO 2) and in addition, one of the Elective Outcomes (LO 3 or LO 4 or LO 5).

### GRADE 12

#### EXAMINATION MARK ALLOCATION

Learning Outcome	Marks	Time
1	120 – 160	80 – 90 minutes
2	60 – 80	30 – 40 minutes
Elective	100	60 minutes
Total	300	3 hours

#### WEIGHTING ACCORDING TO TAXONOMY OF COGNITIVE LEVEL

Level		%
1	Knowledge	10 – 20
2	Routine Procedures	40 – 50
3	Complex Procedures	20 – 30
4	Problem Solving	5 – 10

### DISTRIBUTION OF MARKS FOR CORE LEARNING OUTCOMES

Learning Outcome	Topic	Mark distribution
1	Functions and limits	10 – 30
	Trigonometry	10 – 30
	Differentiation	20 – 40
	Integration	20 – 40
	Drawing functions	20 – 30
	Applications (max/min; rates of change; volume & area)	20 – 30
	<b>Total</b>	<b>120 – 160</b>
2	Real and Complex roots	10 – 20
	Exponents and logarithms	20 – 30
	Absolute Value	20 – 30
	Induction	10 – 20
	<b>Total</b>	<b>60 – 80</b>

### DISTRIBUTION OF MARKS FOR ELECTIVE LEARNING OUTCOMES

Learning Outcome	Topic	Mark distribution
3	Probability	60 – 70
	Descriptive statistics	30 – 40
	<b>Total</b>	<b>100</b>
4	Financial models	40 – 60
	Recursive models	40 – 60
	<b>Total</b>	<b>100</b>
5	Graph theory	40 – 60
	Matrices	40 – 60
	<b>Total</b>	<b>100</b>

## C. INTERPRETATION OF REQUIREMENTS

### INFORMATION BOOKLET

#### Algebra

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$$

$$z = a + bi$$

$$z^* = a - bi$$

$$\ln A + \ln B = \ln(AB)$$

$$\ln A - \ln B = \ln\left(\frac{A}{B}\right)$$

$$\ln A^n = n \ln A$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

#### Calculus

$$\text{Area} = \lim_{n \rightarrow \infty} \left( \frac{b-a}{n} \right) \sum_{i=1}^n f(x_i)$$

$$\int_a^b x^n dx = \left[ \frac{x^{n+1}}{n+1} \right]_a^b$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\int f'(g(x)) \cdot g'(x) dx = f(g(x)) + c$$

$$\int f(x) \cdot g'(x) dx = f(x) \cdot g(x) - \int g(x) \cdot f'(x) dx + c$$

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$$

$$V = \pi \int_a^b y^2 dx$$

Function	Derivative
$x^n$	$nx^{n-1}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \cdot \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cdot \cot x$
$f(g(x))$	$f'(g(x)) \cdot g'(x)$
$f(x) \cdot g(x)$	$g(x) \cdot f'(x) + f(x) \cdot g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

$$A = \frac{1}{2} r^2 \theta$$

$$s = r\theta$$

In  $\triangle ABC$ :

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{Area} = \frac{1}{2} ab \cdot \sin C$$

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$

$$\sin(A \pm B) = \sin A \cdot \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \begin{cases} \cos^2 A - \sin^2 A \\ 2 \cos^2 A - 1 \\ 1 - 2 \sin^2 A \end{cases}$$

$$\sin A \cdot \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\sin A \cdot \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$$

$$\cos A \cdot \cos B = \frac{1}{2} [\cos(A-B) + \cos(A+B)]$$

## Matrix Transformations

$$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

## Finance & Modelling

$$F = P(1+in)$$

$$F = P(1-in)$$

$$F = P(1+i)^n$$

$$F = P(1-i)^n$$

$$F = x \left[ \frac{(1+i)^n - 1}{i} \right]$$

$$P = x \left[ \frac{1 - (1+i)^{-n}}{i} \right]$$

$$r_{eff} = \left( 1 + \frac{r}{k} \right)^k - 1$$

$$P_{n+1} = P_n + rP_n \left( 1 - \frac{P_n}{K} \right)$$

$$R_{n+1} = R_n + aR_n \left( 1 - \frac{R_n}{K} \right) - bR_n F_n$$

$$F_{n+1} = F_n + f b R_n F_n - c F_n$$

## Statistics

$$P(A) = \frac{n(A)}{n(s)}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$${}^n P_r = \frac{n!}{(n-r)!}$$

$${}^n C_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$P(R=r) = \frac{\binom{p}{r} \binom{N-p}{n-r}}{\binom{N}{n}}$$

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}}$$

$$b = \frac{n \sum(xy) - \sum x \sum y}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n(\bar{x})^2}$$

$$b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

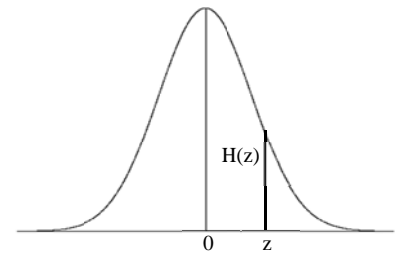
## NORMAL DISTRIBUTION TABLE

Areas under the Normal Curve

$$H(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-\frac{1}{2}x^2} dx$$

$$H(-z) = H(z), H(\infty) = \frac{1}{2}$$

Entries in the table are values of  $H(z)$  for  $z \geq 0$ .



z	,00	,01	,02	,03	,04	,05	,06	,07	,08	,09
0,0	,0000	,0040	,0080	,0120	,0160	,0199	,0239	,0279	,0319	,0359
0,1	,0398	,0438	,0478	,0517	,0557	,0596	,0636	,0675	,0714	,0753
0,2	,0793	,0832	,0871	,0910	,0948	,0987	,1026	,1064	,1103	,1141
0,3	,1179	,1217	,1255	,1293	,1331	,1368	,1406	,1443	,1480	,1517
0,4	,1554	,1591	,1628	,1664	,1700	,1736	,1772	,1808	,1844	,1879
0,5	,1915	,1950	,1985	,2019	,2054	,2088	,2123	,2157	,2190	,2224
0,6	,2257	,2291	,2324	,2357	,2389	,2422	,2454	,2486	,2517	,2549
0,7	,2580	,2611	,2642	,2673	,2704	,2734	,2764	,2794	,2823	,2852
0,8	,2881	,2910	,2939	,2967	,2995	,3023	,3051	,3078	,3106	,3133
0,9	,3159	,3186	,3212	,3238	,3264	,3289	,3315	,3340	,3365	,3389
1,0	,3413	,3438	,3461	,3485	,3508	,3531	,3554	,3577	,3599	,3621
1,1	,3643	,3665	,3686	,3708	,3729	,3749	,3770	,3790	,3810	,3830
1,2	,3849	,3869	,3888	,3907	,3925	,3944	,3962	,3980	,3997	,4015
1,3	,4032	,4049	,4066	,4082	,4099	,4115	,4131	,4147	,4162	,4177
1,4	,4192	,4207	,4222	,4236	,4251	,4265	,4279	,4292	,4306	,4319
1,5	,4332	,4345	,4357	,4370	,4382	,4394	,4406	,4418	,4429	,4441
1,6	,4452	,4463	,4474	,4484	,4495	,4505	,4515	,4525	,4535	,4545
1,7	,4554	,4564	,4573	,4582	,4591	,4599	,4608	,4616	,4625	,4633
1,8	,4641	,4649	,4656	,4664	,4671	,4678	,4686	,4693	,4699	,4706
1,9	,4713	,4719	,4726	,4732	,4738	,4744	,4750	,4756	,4761	,4767
2,0	,4772	,4778	,4783	,4788	,4793	,4798	,4803	,4808	,4812	,4817
2,1	,4821	,4826	,4830	,4834	,4838	,4842	,4846	,4850	,4854	,4857
2,2	,4861	,4864	,4868	,4871	,4875	,4878	,4881	,4884	,4887	,4890
2,3	,48928	,48956	,48983	,49010	,49036	,49061	,49086	,49111	,49134	,49158
2,4	,49180	,49202	,49224	,49245	,49266	,49286	,49305	,49324	,49343	,49361
2,5	,49379	,49396	,49413	,49430	,49446	,49461	,49477	,49492	,49506	,49520
2,6	,49534	,49547	,49560	,49573	,49585	,49598	,49609	,49621	,49632	,49643
2,7	,49653	,49664	,49674	,49683	,49693	,49702	,49711	,49720	,49728	,49736
2,8	,49744	,49752	,49760	,49767	,49774	,49781	,49788	,49795	,49801	,49807
2,9	,49813	,49819	,49825	,49831	,49836	,49841	,49846	,49851	,49856	,49861
3,0	,49865	,49869	,49874	,49878	,49882	,49886	,49889	,49893	,49896	,49900
3,1	,49903	,49906	,49910	,49913	,49916	,49918	,49921	,49924	,49926	,49929
3,2	,49931	,49934	,49936	,49938	,49940	,49942	,49944	,49946	,49948	,49950
3,3	,49952	,49953	,49955	,49957	,49958	,49960	,49961	,49962	,49964	,49965
3,4	,49966	,49968	,49969	,49970	,49971	,49972	,49973	,49974	,49975	,49976
3,5	,49977									
3,6	,49984									
3,7	,49989									
3,8	,49993									
3,9	,49995									
4,0	,49997									